

***CP* violation and family mixing in the effective electroweak Lagrangian**

D. Espriu* and J. Manzano†

Departament d'Estructura i Constituents de la Matèria and IFAE, Universitat de Barcelona, Diagonal, 647, E-08028 Barcelona, Spain

(Received 3 November 2000; published 7 March 2001)

We construct the most general effective Lagrangian of the matter sector of the standard model, including mixing and *CP* violating terms. The Lagrangian contains the effective operators that give the leading contribution in theories where the physics beyond the standard model shows at a scale $\Lambda \gg M_W$. We perform the diagonalization and passage to the physical basis in full generality. We determine the contribution to the different observables and discuss the possible new sources of *CP* violation, the idea being to be able to gain some knowledge about new physics beyond the standard model from general considerations, without having to compute model by model. The values of the coefficients of the effective Lagrangian in some theories, including the standard model, are presented and we try to draw some general conclusions about the general pattern exhibited by physics beyond the standard model in what concerns *CP* violation. In the process we have had to deal with two theoretical problems which are very interesting in their own: the renormalization of the CKM matrix elements and the wave function renormalization in the on-shell scheme when mixing is present.

DOI: 10.1103/PhysRevD.63.073008

PACS number(s): 12.15.Ff, 11.10.Hi, 12.15.Hh

I. INTRODUCTION

The origin of *CP* violation remains, to this date, one of the unsolved puzzles in particle physics. In the minimal standard model there is only one source of *CP* violation, as is well known. Although the most general mass matrix does, in principle, contain a large number of phases, only the left handed diagonalization matrices survive [combined in a single Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix which we denote by K]. This matrix contains only one observable complex phase.

Whether this source of *CP* violation is enough to explain our world is, at present, an open question. In the near future new experimental data (mostly involving third generation quarks) will allow us to measure with good precision those elements of the CKM matrix which are poorly known at present. One of the commonly stated purposes of the new generation of experiments is to check the “unitarity of the CKM matrix.”

Stated this way, the purpose sounds rather meaningless. Of course if one only retains the three known generations mixing occurs through a 3×3 matrix that is, by construction, necessarily unitary. What is really meant by the above statement is whether the observable *S*-matrix elements, which at tree level are proportional to a CKM matrix element, when measured in charged weak decays, turn out to be in good agreement with the tree-level unitarity relations predicted by the standard model. If we write, for instance,

$$\langle q_j | W_\mu^+ | q_i \rangle = \mathcal{U}_{ij} V_\mu. \quad (1)$$

At the tree level, it is clear that $\mathcal{U} = K$ and unitarity of the CKM matrix implies

$$\sum_k \mathcal{U}_{ik} \mathcal{U}_{jk}^* = \delta_{ij}. \quad (2)$$

However, even if there is no new physics at all beyond the standard model radiative corrections contribute to the matrix elements relevant for weak decays and spoil the unitarity of the “CKM matrix” $\mathcal{U}$, in the sense that the corresponding *S*-matrix elements are no longer constrained to verify the above relation. Obviously, departures from unitarity due to the electroweak radiative corrections are bound to be small. Later we shall see at what level are violations of unitarity due to radiative corrections to be expected.

But of course, the violations of unitarity which are really interesting are those caused by new physics. Physics beyond the standard model can manifest itself in several ways and at several scales. In this work we shall adopt the viewpoint that new physics may appear at a scale Λ which is relatively large compared to the M_Z scale. This remark includes the scalar sector too; i.e. we assume that the Higgs particle—if it exists at all—is sufficiently heavy. If this is so, an expansion in inverse powers of Λ is justified and effective Lagrangian techniques [1] can be used. The scale Λ could, for instance, be the mass of a new heavy fermion, some compositeness scale, or simply the Higgs boson mass.

It is particularly interesting, at least from an instructive point of view, to consider the case of a new heavy generation. We can proceed in two ways. One possibility is to treat all fermions, light or heavy, on the same footing. We would then end up with a 4×4 unitary mixing matrix, the one corresponding to the light quarks being a 3×3 submatrix which, of course need not be—and in fact, will not be—unitary. Stated this way the departures from unitarity (already at tree level) could conceivably be sizeable. The alternative way to proceed would be, in the philosophy of effective Lagrangians, to integrate out completely the heavy generation. One is then left, at lowest order in the inverse mass expansion, with just the ordinary kinetic and mass terms for light quarks, leading—obviously—to an ordinary

*Email address: espriu@ecm.ub.es

†Email address: manzano@ecm.ub.es

3×3 mixing matrix, which is of course unitary. Naturally, there is no logical contradiction between the two procedures because what really matters is the physical S -matrix element and this gets, if we follow the second procedure (integrating out the heavy fields), two type of contributions: from the lowest dimensional operators involving only light fields and from the additional operators obtained after integrating out the heavy fields. The result for the observable S -matrix element should obviously be the same whatever procedure we follow, but using the second method we learn that the violations of unitarity in the (three generation) unitarity triangle are suppressed by some heavy mass (since an additional generation decouples in the observables we are interested [2]). This simple consideration illustrates the virtues of the effective Lagrangian approach. We shall say more about this later.

The purpose of this paper is to use the philosophy behind effective Lagrangians to try and learn some more insight on the issue of possible sources of CP violation beyond the standard model. We shall, in particular, determine the most general parametrization, to the lowest non-trivial order, of all possible family mixing and CP -violating effects in the matter sector of the standard model. (Of course, being completely general is impossible, so some restrictions shall apply to our considerations. These shall be spelled out in Sec. II.)

According to our philosophy we shall, first of all, classify all possible operators of lowest dimensionality which, respecting all the appropriate symmetries, can be added to the ones which are present in the minimal standard model. Then we shall analyze the most general kinetic and mass terms (including, obviously, mixing). Even these terms may already be different from those in the minimal standard model, the reason being that some field redefinitions which are routinely done in the standard model are not innocuous in more general models. We then proceed to diagonalize both, the mass and kinetic terms, and determine the effects of the diagonalization procedure, i.e. of passing to the physical basis, on the most general set of operators of dimension four (again including the possibility of off-diagonal couplings in family space). We then discuss the conditions for these operators to be CP -odd.

Note that in the minimal standard model, only the left-handed diagonalization matrices appear in physical processes (combined in the CKM matrix K). When operators beyond the standard model are included (originally written in the basis of weak eigenstates) the passage to the physical (diagonal) basis becomes more involved. Operators involving just left handed fields transform into more complex structures involving K and redefined effective couplings. These structures were not present before the change of variables because, in the weak eigenstates basis, they explicitly break $SU(2)_L$. For operators involving right handed fields the situation is different. We will show that passing to the physical basis amounts only to a redefinition of their couplings, without changing their structure. It comes perhaps as a surprise that beyond the standard model the passage to the physical basis involves in either case *non-unitary* matrices.

One of the major contributions presented in this work is the detailed treatment of the issue of wave-function and CKM matrix elements renormalization constants. There are

two reasons to do so. On the one hand, contact with physical matrix elements requires that the external legs are properly normalized and there is *a priori* no reason why new physics cannot contribute to the wave-function renormalization constants, exactly as they do to the effective vertices. It is simply inconsistent to include one and not the other. In fact, in the case of the CKM matrix elements, their renormalization turns out to be related to the wave-function renormalization matrices, so it is obviously necessary to deal with this issue one way or another even in the standard model. On the other hand, it must be said that the actual on-shell prescription to incorporate the wave-function renormalization conditions is not fully understood yet when mixing is present. This provides for us a second motivation to treat this problem carefully.

Another motivation to present the effective Lagrangian analysis of the family mixing and CP violation problems is that it can be applied to an analysis of radiative corrections (for instance in the minimal standard model itself) through the use of effective couplings. For a particular process the leading contribution from radiative corrections comes as a redefinition of the effective couplings, i.e. to some specific values for the coefficients of the effective Lagrangian. Once determined, they can be used for other observables without needing to compute them anew. This procedure proved to be very efficient in recent years in the context of CERN e^+e^- collider LEP physics and neutral currents phenomenology [3].

Finally and somewhat related to the previous issue is the fact that an effective Lagrangian provides a convenient book-keeping device to treat deviations with respect to the standard model tree level predictions in a particular process. Questions like whether is it legitimate or not to use the unitarity of the CKM matrix in a given process, given that one is precisely looking for violations of unitarity, can be posed and answered systematically in an effective Lagrangian framework.

The paper is organized in the following way. In the next section we extend the effective electroweak Lagrangian in the matter sector [4] to the case where there is mixing amongst different generations. We shall see which restrictions CP -conservation imposes on the coefficients of the effective Lagrangian. We shall then discuss in Sec. III the passage to the physical basis, which is quite interesting in the present framework, and is in fact one of the main results of this work. The effective couplings and some possible observable effects are discussed in Sec. IV. In Sec. V we shall take into account the effects due to renormalization, comment the expected size of the standard model radiative corrections and point out some open problems. In Sec. VI we shall briefly consider two examples: a heavy doublet and the standard model with a heavy Higgs boson. Conclusions shall be summarized in Sec. VII.

II. EFFECTIVE LAGRANGIAN

Let us first state the assumptions behind the present framework. We shall assume that the scale of any new physics beyond the standard model is sufficiently high so that an

inverse mass expansion is granted, and we shall organize the effective Lagrangian accordingly. We shall also assume that the Higgs field either does not exist or is massive enough to permit an effective Lagrangian treatment by expanding in inverse powers of its mass, M_H . In short, we assume that all as yet undetected new particles are heavy, with a mass much larger than the energy scale at which the effective Lagrangian is to be used. Thus it is natural to use a non-linear realization of the $SU(2)_L \times U(1)_Y$ symmetry where the unphysical scalar fields are collected in a unitary 2×2 matrix U (see e.g. [1]).

An additional assumption that we may make at some point is that, whatever is the source of CP -violation beyond the standard model, when compared to the CP conserving part, is “small.” This statement does need qualification. What really matters, of course, is the observable value of the CP violating parameters, which are customarily calculated in the mass eigenstate basis. On the other hand, new physics may (or may not, we do not know for sure) appear naturally in the weak basis; i.e. with fields transforming as irreducible representations of the gauge group. When operators beyond the standard model are included they will have, in general, a CP -violating and a CP -conserving part when written in the weak basis. For the sake of discussion let us imagine an scenario where the origin of fermion masses is unrelated with the physics that contributes to effective operators beyond those already contained in the standard model (perhaps because the former is associated to a very large scale). Then new physics can be separated somehow in two parts: one part contributes to the kinetic and Yukawa operators in the weak basis and is responsible for the known mass structure of the matter sector; the other part contributes, again in the weak basis, to a set of effective operators [the one described later by Eqs. (8)]. If we assume, for example, that the latter are totally or almost CP conserving then can have the peculiar situation that many CP -violating phases may appear in the coefficients of the effective operators when we pass to the physical base; phases which would not be observable in the minimal standard model. In short, it is conceivable that CP conserving physics triggers CP -violation in the physical basis. Of course the converse is theoretically also possible, CP violating phases may disappear once things are written in the physical basis.

Let us commence our classification of the operators present in the matter sector of the effective electroweak Lagrangian. We shall use the following projectors:

$$R = \frac{1 + \gamma^5}{2}, \quad L = \frac{1 - \gamma^5}{2}, \quad \tau^u = \frac{I + \tau^3}{2}, \quad \tau^d = \frac{I - \tau^3}{2}, \quad (3)$$

where R is the right projector and L the left projector in chirality space, and τ^u is the up projector and τ^d the down projector in $SU(2)$ space. The different gauge groups act on the scalar, U , and fermionic, f_L, f_R , fields in the following way:

$$\begin{aligned} D_\mu U &= \partial_\mu U + ig \frac{\tau}{2} \cdot W_\mu U - ig' U \frac{\tau^3}{2} B_\mu, \\ D_\mu^L f_L &= \partial_\mu f_L + ig \frac{\tau}{2} \cdot W_\mu f_L + ig' \left(Q - \frac{\tau^3}{2} \right) B_\mu f_L \\ &\quad + ig_s \frac{\lambda}{2} \cdot G_\mu f_L, \\ D_\mu^R f_R &= \partial_\mu f_R + ig' Q B_\mu f_R + ig_s \frac{\lambda}{2} \cdot G_\mu f_R. \end{aligned} \quad (4)$$

The following terms are universal. They must be present in any effective theory whose long-distance properties are those of the standard model. They correspond to the standard model kinetic and mass terms (we use the notation f to describe both left and right degrees of freedom simultaneously)

$$\begin{aligned} \mathcal{L}_{kin}^L &= i \bar{f} X_L \gamma^\mu D_\mu^L f, \\ \mathcal{L}_{kin}^R &= i \bar{f} (\tau^\mu X_{Ru} + \tau^d X_{Rd}) \gamma^\mu D_\mu^R f, \\ \mathcal{L}_m &= - \bar{f} (U (\tau^\mu \tilde{y}_u^f + \tau^d \tilde{y}_d^f) R + (\tau^\mu \tilde{y}_u^{f\dagger} + \tau^d \tilde{y}_d^{f\dagger}) U^\dagger L) f. \end{aligned} \quad (5)$$

X_L , X_{Ru} and X_{Rd} are non-singular Hermitian matrices having only family indices, and $\tilde{y}_u^f$ and $\tilde{y}_d^f$ are arbitrary matrices and have only family indices too. Note that in general $X_{Rd} \neq X_{Ru}$, as the only restriction is gauge invariance. In the standard model, these matrices can always be reabsorbed by an appropriate redefinition of the fields (we shall see this explicitly later), so one does not even contemplate the possibility that left and right “kinetic” terms are differently normalized, but this is perfectly possible in an effective theory, and the transformations required to bring these kinetic terms to the standard form do leave some fingerprints.

In order to write the above terms in the familiar form in the standard model we shall perform a series of chiral changes of variables. In general, due to the axial anomaly, these changes will modify the CP violating terms

$$\mathcal{L}_\theta = \epsilon^{\alpha\beta\mu\nu} (\theta_1 B_{\alpha\beta} B_{\mu\nu} + \theta_2 W_{\alpha\beta}^a W_{\mu\nu}^a + \theta_3 G_{\alpha\beta}^a G_{\mu\nu}^a), \quad (6)$$

but we will not care about that here.

Notice the appearance of the unitary matrix U collecting the (unphysical) Goldstone bosons. The Higgs field—as emphasized above—should it exist, has been integrated out. Since the global symmetries are non-linearly realized the above Lagrangian is not renormalizable.

In addition to Eq. (5) a number of operators of dimension four should be included in the matter sector of the effective electroweak Lagrangian. They are, to begin with, necessary as counterterms to remove some ultraviolet divergences that appear at the quantum level due to the non-linear nature of Eq. (5). Moreover, physics beyond the standard model does in general contribute to the coefficients of those operators, as it may do to X_L , X_{Ru} , X_{Rd} , $\tilde{y}_u$ and $\tilde{y}_d$. The dimension 4 operators can be written generically as

$$\mathcal{L}_L = \bar{f} \gamma_\mu M_L O_L^\mu L f + \text{H.c.}, \quad (7)$$

$$\mathcal{L}_R = \bar{f} \gamma_\mu M_R O_R^\mu R f + \text{H.c.},$$

where M_L and M_R are matrices having family indices only and O_L^μ and O_R^μ are operators of dimension one having weak indices (u,d) only. These operators were first written by [4] in the case where mixing between families is absent. They have been recently considered in [5] and [6]. The extension to the three-generation case is new.

The complete list of the dimension four operators is

$$\begin{aligned} \mathcal{L}_L^1 &= i \bar{f} M_L^1 \gamma^\mu U (D_\mu U)^\dagger L f + \text{H.c.}, \\ \mathcal{L}_L^2 &= i \bar{f} M_L^2 \gamma^\mu (D_\mu U) \tau^3 U^\dagger L f + \text{H.c.}, \\ \mathcal{L}_L^3 &= i \bar{f} M_L^3 \gamma^\mu U \tau^3 U^\dagger (D_\mu U) \tau^3 U^\dagger L f + \text{H.c.}, \\ \mathcal{L}_L^4 &= i \bar{f} M_L^4 \gamma^\mu U \tau^3 U^\dagger D_\mu^L L f + \text{H.c.}, \\ \mathcal{L}_R^1 &= i \bar{f} M_R^1 \gamma^\mu U^\dagger (D_\mu U) R f + \text{H.c.}, \\ \mathcal{L}_R^2 &= i \bar{f} M_R^2 \gamma^\mu \tau^3 U^\dagger (D_\mu U) R f + \text{H.c.}, \\ \mathcal{L}_R^3 &= i \bar{f} M_R^3 \gamma^\mu \tau^3 U^\dagger (D_\mu U) \tau^3 R f + \text{H.c.} \end{aligned} \quad (8)$$

Without any loss of generality we take the matrices in family space M_L^1 , M_R^1 , M_L^3 and M_R^3 Hermitian, while M_L^2 , M_R^2 and M_L^4 are completely general. If we require the above operators to be CP conserving, the matrices $M_{L,R}^i$ must be real (see Sec. IV).

In addition to the above ones, physics beyond the standard model generates, in general, an infinite tower of higher-dimensional operators with $d \geq 5$ [these operators are eventually required as counterterms too due to the non-linear nature of the Lagrangian (5)]. On dimensional grounds these operators shall be suppressed by powers of the scale Λ characterizing new physics or by powers of $4\pi v$ (v being the scale of the breaking—250 GeV). Therefore, if the scale of new physics is sufficiently high the contribution of higher dimensional operators can be neglected as compared to those of $d=4$. Of course for this to be true the later must be non-vanishing and sizeable. Thanks to the violation of the Appelquist-Carazzone decoupling theorem [7] in spontaneously broken theories, this is often the case, unless the new physics is tuned so as to be decoupling as is the case in the minimal supersymmetric standard model (see e.g. [8] for a recent discussion on this matter).

III. PASSAGE TO THE PHYSICAL BASIS

Let us first consider the operators which are already present in the standard model, Eq. (5). The diagonalization and passage to the physical basis are of course well known, but some modifications are required when one considers the general case in Eq. (5) so it is worth going through the discussion with some detail.

We perform first the unitary change of variables

$$f = [\tilde{V}_L L + (\tilde{V}_{Ru} \tau^\mu + \tilde{V}_{Rd} \tau^d) R] f, \quad (9)$$

with the help of the unitary matrices $\tilde{V}_L$, $\tilde{V}_{Ru}$ and $\tilde{V}_{Rd}$. Hence

$$(\tilde{y}_u^f \tau^\mu + \tilde{y}_d^f \tau^d) \rightarrow (\tilde{V}_L^\dagger \tilde{y}_u^f \tilde{V}_{Ru} \tau^\mu + \tilde{V}_L^\dagger \tilde{y}_d^f \tilde{V}_{Rd} \tau^d), \quad (10)$$

and

$$X_L \rightarrow \tilde{V}_L^\dagger X_L \tilde{V}_L = D_L,$$

$$X_{Ru} \rightarrow \tilde{V}_{Ru}^\dagger X_{Ru} \tilde{V}_{Ru} = D_{Ru}, \quad (11)$$

$$X_{Rd} \rightarrow \tilde{V}_{Rd}^\dagger X_{Rd} \tilde{V}_{Rd} = D_{Rd},$$

where D_L , D_{Ru} and D_{Rd} are diagonal matrices with eigenvalues different from zero. Then, with the help of the non-unitary transformation

$$f \rightarrow [D_L^{-1/2} L + (D_{Ru}^{-1/2} \tau^\mu + D_{Rd}^{-1/2} \tau^d) R] f, \quad (12)$$

we obtain

$$\begin{aligned} D_L &\rightarrow (D_L^{-1/2})^* D_L D_L^{-1/2} = I, \\ D_{Ru}^R &\rightarrow (D_{Ru}^{-1/2})^* D_{Ru} D_{Ru}^{-1/2} = I, \end{aligned} \quad (13)$$

$$D_{Rd}^R \rightarrow (D_{Rd}^{-1/2})^* D_{Rd} D_{Rd}^{-1/2} = I,$$

and the matrix $\tilde{y}_u^f \tau^\mu + \tilde{y}_d^f \tau^d$ transforms into

$$\begin{aligned} (D_L^{-1/2})^* \tilde{V}_L^\dagger \tilde{y}_u^f \tilde{V}_{Ru} D_{Ru}^{-1/2} \tau^\mu + (D_L^{-1/2})^* \tilde{V}_L^\dagger \tilde{y}_d^f \tilde{V}_{Rd} D_{Rd}^{-1/2} \tau^d \\ \equiv y_u^f \tau^\mu + y_d^f \tau^d, \end{aligned} \quad (14)$$

where y_u^f and y_d^f are the Yukawa couplings. Thus, the left and right kinetic terms can be brought to the canonical form at the sole expense of redefining the Yukawa couplings. Since this is all there is in the standard model, we see that the effect of considering the more general coefficients for the kinetic terms is irrelevant. This will not be the case when additional operators are considered. Fermions transform, up to this point, in irreducible representations of the gauge group.

We now perform the unitary change of variables

$$f \rightarrow [(V_{Lu} \tau^\mu + V_{Ld} \tau^d) L + (V_{Ru} \tau^\mu + V_{Rd} \tau^d) R] f, \quad (15)$$

with unitary matrices V_{Lu} , V_{Ru} , V_{Ld} and V_{Rd} and having family indices only. They are chosen so that the Yukawa terms become diagonal and definite positive (see e.g. [9])

$$\begin{aligned} (V_{Lu}^\dagger \tau^\mu + V_{Ld}^\dagger \tau^d) (y_u^f \tau^\mu + y_d^f \tau^d) (V_{Ru} \tau^\mu + V_{Rd} \tau^d) \\ = d_u^f \tau^\mu + d_d^f \tau^d. \end{aligned} \quad (16)$$

After all these transformations $\mathcal{L}_m$ transforms into

$$\mathcal{L}_m = -\bar{f}\{(\tau^\mu U + K^\dagger \tau^d U)\tau^\mu d_u^f + (\tau^d U + K \tau^\mu U)\tau^d d_d^f\}Rf + \text{H.c.}, \quad (17)$$

where $K \equiv V_{Lu}^\dagger V_{Ld}$ is well known Cabibbo-Kobayashi-Maskawa matrix. Note in Eq. (17) that when we set $U=I$ we obtain

$$\mathcal{L}_m = -\bar{f}(\tau^\mu d_u^f + \tau^d d_d^f)Rf + \text{H.c.}, \quad (18)$$

which is a diagonal mass term. Fermions now transform in *reducible* representations of the gauge group.

The left and right kinetic terms now read

$$\mathcal{L}_{kin}^R = i\bar{f}\gamma^\mu D_\mu^R Rf, \quad (19)$$

and

$$\begin{aligned} \mathcal{L}_{kin}^L = i\bar{f}\gamma^\mu L \left\{ \partial_\mu + ig' \left(Q - \frac{\tau^3}{2} \right) B_\mu + ig \frac{\tau^3}{2} W_\mu^3 \right. \\ \left. + ig \left(K \frac{\tau^-}{2} W_\mu^+ + K^\dagger \frac{\tau^+}{2} W_\mu^- \right) + ig_s \frac{\lambda}{2} \cdot G_\mu \right\} f. \end{aligned} \quad (20)$$

CP violation is present if and only if $K \neq K^*$.

As is well known, some freedom for additional phase redefinitions is left. If we make the replacement

$$f \rightarrow [(W_{Lu}\tau^\mu + W_{Ld}\tau^d)L + (W_{Ru}\tau^\mu + W_{Rd}\tau^d)R]f, \quad (21)$$

we have to change

$$K = V_{Lu}^\dagger V_{Ld} \rightarrow W_{Lu}^\dagger V_{Lu}^\dagger V_{Ld} W_{Ld} = W_{Lu}^\dagger K W_{Ld}, \quad (22)$$

and

$$d_u = V_{Lu}^\dagger y_u^f V_{Ru} \rightarrow W_{Lu}^\dagger V_{Lu}^\dagger y_u^f V_{Ru} W_{Ru} = W_{Lu}^\dagger d_u^f W_{Ru}, \quad (23)$$

$$d_d = V_{Ld}^\dagger y_d^f V_{Rd} \rightarrow W_{Ld}^\dagger V_{Ld}^\dagger y_d^f V_{Rd} W_{Rd} = W_{Ld}^\dagger d_d^f W_{Rd},$$

but if we want to keep d_u^f and d_d^f diagonal real and definite positive, and if we suppose that they do not have degenerate eigenstates the only possibility for the unitary matrices W is to be diagonal with $W_{R(u,d)} = W_{L(u,d)}$. This freedom can be used, for example, to extract five phases from K . After this no further redefinitions are possible neither in the left nor in the right handed sector.

So much for the standard model. Let us now move to the more general case represented at low energies by the $d=4$ operators listed in the previous section. We have to analyze the effect of the transformations given by Eqs. (9), (12) and (15) [here we include in Eq. (15) the effect of Eq. (21)] on the operators (8). The composition of those transformations is given by

$$\begin{aligned} f \rightarrow \tilde{V}_L (D^L)^{-1/2} (V_{Lu}\tau^\mu + V_{Ld}\tau^d)Lf + (\tilde{V}_{Ru}(D_u^R)^{-1/2}V_{Ru}\tau^\mu \\ + \tilde{V}_{Rd}(D_d^R)^{-1/2}V_{Rd}\tau^d)Rf \\ \equiv (C_L^u\tau^\mu + C_L^d\tau^d)Lf + (C_R^u\tau^\mu + C_R^d\tau^d)Rf. \end{aligned} \quad (24)$$

Note that because of the presence of the matrices D , the matrices C are in general non-unitary. We begin with the effective operators involving left handed fields. In this case when we perform transformation (24) we obtain

$$\mathcal{L}_L \rightarrow \bar{f}\gamma_\mu \mathcal{O}_L^\mu Lf + \text{H.c.}, \quad (25)$$

with the operator $\mathcal{O}_L^\mu$ containing family and weak indices given by

$$\begin{aligned} \mathcal{O}_L^\mu = N\tau^\mu O_L^\mu \tau^\mu + NK\tau^\mu O_L^\mu \tau^d + K^\dagger NK\tau^d O_L^\mu \tau^d \\ + K^\dagger N\tau^d O_L^\mu \tau^\mu, \end{aligned} \quad (26)$$

where we have defined

$$N \equiv C_L^{u\dagger} M_L C_L^u. \quad (27)$$

Thus new structures do appear involving the CKM matrix K and left-handed fields. The former cannot be reduced to our starting set of operators by a simple redefinition of the original couplings M_L .

The case of the effective operators involving right handed fields ($\mathcal{L}_R$) is, in this sense, simpler because transformation (24) only redefine the matrices M_R . The operators involving right-handed fields are

$$\mathcal{L}_R^p = i\bar{f}\gamma_\mu M_R^p O_p^\mu Rf + \text{H.c.}, \quad (28)$$

with

$$\begin{aligned} O_1^\mu = U^\dagger(D_\mu U), \quad O_2^\mu = \tau^3 U^\dagger(D_\mu U), \\ O_3^\mu = \tau^3 U^\dagger(D_\mu U)\tau^3. \end{aligned} \quad (29)$$

Note that because of the H.c. in $\mathcal{L}_R^p$ we can change O_2^μ by $U^\dagger(D_\mu U)\tau^3$ along with M_R^2 by $M_R^{2\dagger}$. So under the transformation (24) we obtain

$$\mathcal{L}_R^p \rightarrow i\bar{f}\gamma_\mu \mathcal{O}_{pR}^\mu Rf + \text{H.c.},$$

with the operators $\mathcal{O}_{pR}^\mu$ containing family and weak indices given by

$$\begin{aligned} \mathcal{O}_{pR}^\mu = C_R^{u\dagger} M_R^p C_R^u \tau^\mu O_p^\mu \tau^\mu + C_R^{u\dagger} M_R^p C_R^d \tau^\mu O_p^\mu \tau^d \\ + C_R^{d\dagger} M_R^p C_R^u \tau^d O_p^\mu \tau^\mu + C_R^{d\dagger} M_R^p C_R^d \tau^d O_p^\mu \tau^d, \end{aligned} \quad (30)$$

hence

$$\begin{aligned} \sum_{p=1}^3 \mathcal{L}_R^p \rightarrow \sum_{p=1}^3 (i\bar{f}\gamma_\mu \mathcal{O}_{pR}^\mu Rf + \text{H.c.}) \\ = \sum_{p=1}^3 (i\bar{f}\gamma_\mu \tilde{M}_R^p O_p^\mu Rf + \text{H.c.}), \end{aligned} \quad (31)$$

with

$$\begin{aligned}\tilde{M}_R^1 &= C_+^\dagger M_R^1 C_+ + C_-^\dagger M_R^2 C_+ + C_-^\dagger M_R^3 C_-, \\ \tilde{M}_R^2 &= C_-^\dagger M_R^1 C_+ + C_+^\dagger M_R^2 C_+ + C_+^\dagger M_R^3 C_-, \\ \tilde{M}_R^3 &= C_-^\dagger M_R^1 C_- + C_+^\dagger M_R^2 C_- + C_+^\dagger M_R^3 C_+, \end{aligned} \quad (32)$$

where $C_\pm = (C_R^u \pm C_R^d)/2$. Hence, transformations (24) can be absorbed by a mere redefinition of the matrices M_R^1 , M_R^2 and M_R^3 .

IV. EFFECTIVE COUPLINGS AND CP VIOLATION

After the transformations discussed in the previous section we are now in the physical basis and in a position to discuss the physical relevance of the couplings in the effective Lagrangian. On dimensional grounds the contribution of all possible dimension four operators to the vertices can be parametrized in terms of effective couplings (see e.g. [10])

$$\begin{aligned}\mathcal{L}_{eff} &= -g_s \bar{f} \gamma^\mu (a_L L + a_R R) \lambda \cdot G_\mu f - e \bar{f} \gamma^\mu (b_L L + b_R R) A_\mu f \\ &\quad - \frac{e}{2c_W s_W} \bar{f} \gamma^\mu (g_L L + g_R R) Z_\mu f - \frac{e}{s_W} \bar{f} \gamma^\mu \\ &\quad \times (h_L L + h_R R) \frac{\tau^-}{2} W_\mu^+ f - \frac{e}{s_W} \bar{f} \gamma^\mu (h_L^\dagger L + h_R^\dagger R) \frac{\tau^+}{2} W_\mu^- f, \end{aligned} \quad (33)$$

where we define

$$\begin{aligned}a_{LR} &= a_{LR}^u \tau^u + a_{LR}^d \tau^d, \quad b_{LR} = b_{LR}^u \tau^u + b_{LR}^d \tau^d, \\ g_{LR} &= g_{LR}^u \tau^u + g_{LR}^d \tau^d. \end{aligned} \quad (34)$$

After rewriting the effective operators (8) in the physical basis, their contribution to the couplings $a_R, a_L, b_R, \dots$ can be found out by setting $U=I$.

The operators involving right-handed fields give rise to ($c_W = g/\sqrt{g^2 + g'^2}$ and $s_W = g'/\sqrt{g^2 + g'^2}$ are the cosine and sine of the Weinberg angle, respectively)

$$\begin{aligned}\sum_{p=1}^3 \mathcal{L}_R^p &= -\bar{f} \gamma^\mu (\tilde{M}_R^1 + \tilde{M}_R^2 \tau^3) \left[\frac{e}{s_W} \left(\frac{\tau^-}{2} W_\mu^+ + \frac{\tau^+}{2} W_\mu^- \right) \right. \\ &\quad \left. + \frac{e}{c_W s_W} \frac{\tau^3}{2} Z_\mu \right] R f - \bar{f} \gamma^\mu \tilde{M}_R^3 \tau^3 \left[\frac{e}{s_W} \left(\frac{\tau^-}{2} W_\mu^+ \right. \right. \\ &\quad \left. \left. + \frac{\tau^+}{2} W_\mu^- \right) + \frac{e}{c_W s_W} \frac{\tau^3}{2} Z_\mu \right] \tau^3 R f + \text{H.c.} \end{aligned} \quad (35)$$

For the operators involving left-handed fields we have instead

$$\begin{aligned}\mathcal{L}_L^1 &= \bar{f} \gamma^\mu \left\{ \frac{e}{c_W s_W} \left(N^1 \frac{\tau^u}{2} - K^\dagger N^1 K \frac{\tau^d}{2} \right) Z_\mu + \frac{e}{s_W} \left(N^1 K \frac{\tau^-}{2} W_\mu^+ \right. \right. \\ &\quad \left. \left. + K^\dagger N^1 \frac{\tau^+}{2} W_\mu^- \right) \right\} L f + \text{H.c.}, \end{aligned} \quad (36)$$

$$\begin{aligned}\mathcal{L}_L^2 &= -\bar{f} \gamma^\mu \left\{ \frac{e}{c_W s_W} \left(N^2 \frac{\tau^u}{2} + K^\dagger N^2 K \frac{\tau^d}{2} \right) Z_\mu \right. \\ &\quad \left. + \frac{e}{s_W} \left(-N^2 K \frac{\tau^-}{2} W_\mu^+ + K^\dagger N^2 \frac{\tau^+}{2} W_\mu^- \right) \right\} L f + \text{H.c.}, \end{aligned} \quad (37)$$

$$\begin{aligned}\mathcal{L}_L^3 &= -\bar{f} \gamma^\mu \left\{ \frac{e}{c_W s_W} \left(N^3 \frac{\tau^u}{2} - K^\dagger N^3 K \frac{\tau^d}{2} \right) Z_\mu \right. \\ &\quad \left. + \frac{e}{s_W} \left(-N^3 K \frac{\tau^-}{2} W_\mu^+ - K^\dagger N^3 \frac{\tau^+}{2} W_\mu^- \right) \right\} L f + \text{H.c.} \end{aligned} \quad (38)$$

The contribution from $\mathcal{L}_L^4$ is a little bit different and deserves some additional comments. Let us first see how this effective operator looks in the physical basis and after setting $U=I$

$$\begin{aligned}\mathcal{L}_L^4 &= -\bar{f} \gamma^\mu \left\{ (N^4 \tau^u - K^\dagger N^4 K \tau^d) \left[-i \partial_\mu + e Q A_\mu \right. \right. \\ &\quad \left. \left. + \frac{e}{c_W s_W} \left(\frac{\tau^3}{2} - Q s_W^2 \right) Z_\mu + g_s \frac{\lambda}{2} \cdot G_\mu \right] + \frac{e}{s_W} \left(N^4 K \frac{\tau^-}{2} W_\mu^+ \right. \right. \\ &\quad \left. \left. - K^\dagger N^4 \frac{\tau^+}{2} W_\mu^- \right) \right\} L f + \text{H.c.} \end{aligned} \quad (39)$$

One sees that $\mathcal{L}_L^4$ is the only operator potentially contributing to the gluon and photon effective couplings. This is of course surprising since both the photon and the gluon are associated to currents which are exactly conserved and radiative corrections (including those from new physics) are prohibited at zero momentum transfer. However one should note that the effective couplings listed in Eq. (33) are not directly observable yet because one must take into account the renormalization of the external legs. In fact $\mathcal{L}_L^4$ is the only operator that can possibly contribute to such renormalization at the order we are working. This issue will be discussed in detail in the next section. When the contribution from the external legs is taken into account one observes that $\mathcal{L}_L^4$ can be eliminated altogether from the neutral gauge bosons couplings (and this includes the Z couplings where the conserved current argument does not apply).

Another way of seeing this (as pointed out in [5]) is by realizing that after use of the equations of motion $\mathcal{L}_L^4$ transforms into a Yukawa term, so the effect of $\mathcal{L}_L^4$ can be absorbed by a redefinition of the fermion masses and the CKM matrix, if the fermions are on-shell, as it will be the case in the present discussion. Then it is clear that $\mathcal{L}_L^4$ may possibly

contribute to the renormalization of the CKM matrix elements only (i.e. to the charged current sector).

All this considered, from Eqs. (33) and (35)–(39), and from the results presented in the next section concerning wave function renormalization, we obtain for the photon and gluon couplings the following contribution coming from the effective operators (8):

$$a_L = a_R = b_L = b_R = 0, \quad (40)$$

both for the up and down components. For the Z couplings the contribution of effective operators (8) is

$$\begin{aligned} g_L^u &= -N^1 - N^{1\dagger} + N^{2\dagger} + N^2 + N^3 + N^{3\dagger}, \\ g_L^d &= K^\dagger (N^1 + N^{1\dagger} + N^{2\dagger} + N^2 - N^3 - N^{3\dagger}) K, \\ g_R^u &= \tilde{M}_R^1 + \tilde{M}_R^{1\dagger} + \tilde{M}_R^2 + \tilde{M}_R^{2\dagger} + \tilde{M}_R^3 + \tilde{M}_R^{3\dagger}, \\ g_R^d &= \tilde{M}_R^2 + \tilde{M}_R^{2\dagger} - \tilde{M}_R^1 - \tilde{M}_R^{1\dagger} - \tilde{M}_R^3 - \tilde{M}_R^{3\dagger}. \end{aligned} \quad (41)$$

The contribution from wave-function renormalization cancels the dependence from the vertices on the Hermitian combination $N^4 + N^{4\dagger}$, which is the only one that appears from the vertices themselves.

As for the effective W couplings we give next the contribution coming from the vertices contained in Eq. (8) only. Naturally, in order to get the full effective couplings one must still add the contribution from wave-function renormalization and from the renormalization of the CKM matrix elements induced by Eq. (8). Actually we will see in Sec. VD that these contributions cancel each other at tree level so in fact the following results include the full dependence on N^4

$$\begin{aligned} h_L &= (-N^1 - N^{1\dagger} + N^2 - N^{2\dagger} - N^3 - N^{3\dagger} + N^4 - N^{4\dagger}) K, \\ h_R &= (\tilde{M}_R^1 + \tilde{M}_R^{1\dagger} + \tilde{M}_R^2 - \tilde{M}_R^{2\dagger} - \tilde{M}_R^3 - \tilde{M}_R^{3\dagger}). \end{aligned} \quad (42)$$

The above effective couplings thus summarize all effects due to the mixing of families in the low energy theory caused by the presence of new physics at some large scale Λ . Let us now investigate the possible new sources of CP violation in the above effective couplings.

Generically we can write

$$\mathcal{L}_L = \bar{f} \gamma_\mu S^\mu L f + \text{H.c.}, \quad (43)$$

where

$$\begin{aligned} S^\mu &\equiv N \tau^\mu O^\mu \tau^\mu + N K \tau^\mu O^\mu \tau^d + K^\dagger N K \tau^d O^\mu \tau^d \\ &\quad + K^\dagger N \tau^d O^\mu \tau^\mu, \end{aligned} \quad (44)$$

then under CP we have

$$\mathcal{L}_L \rightarrow \bar{f} \gamma_\mu S'^\mu L f, \quad (45)$$

with

$$\begin{aligned} S'^\mu &\equiv N^T \tau^\mu O^\mu \tau^\mu + K^T N^T \tau^d O^\mu \tau^\mu + K^T N^T K^* \tau^d O^\mu \tau^d \\ &\quad + N^T K^* \tau^\mu O^\mu \tau^d, \end{aligned} \quad (46)$$

so in order to have CP invariance we require

$$N = N^*,$$

$$N K = N K^*,$$

$$K^T N K^* = K^\dagger N K, \quad (47)$$

which can be fulfilled requiring

$$N = N^*, \quad K = K^*. \quad (48)$$

Note that this last condition is sufficient but not necessary, however if we ask for CP invariance of the complete Lagrangian (as we should) the last condition is both sufficient and necessary. Analogously, the right-handed contribution, given by Eq. (35), is CP invariant provided

$$\tilde{M}_R^p = \tilde{M}_R^{p*}. \quad (49)$$

Equations (40), (41) and (42) thus summarize the contribution from dimension four operators to the observables. In addition there will be contributions from other higher dimensional operators, such as for instance dimension five ones (magnetic moment-type operators for example). We expect these to be small in theories such as the ones we are considering here. The reason is that we assume a large mass gap between the energies at which our effective Lagrangian is going to be used and the scale of new physics. This automatically suppresses the contribution of higher dimensional operators. However, non-decoupling effects may be left in dimension four operators, which may depend logarithmically in the scale of the new physics. The clearest example of this is the standard model itself. Since the Higgs boson is there an essential ingredient in proving the renormalizability of the theory, removing it induces new divergences which eventually manifest themselves as logarithms of the Higgs boson mass. This enhances (for a relatively heavy Higgs boson) the importance of the $d=4$ coefficients, albeit in the standard model they are small (except for the top) nonetheless since the $\log M_H^2/M_W^2$ is preceded by a prefactor $y^2/16\pi^2$, where y is a Yukawa coupling (see [5]).

Apart from the issue of wave-function and CKM renormalization, to which we shall turn next, we have finished our theoretical analysis and we can start drawing some conclusions.

One of the first things one observes is that there are no anomalous photon or gluon couplings, diagonal or not in flavor. This excludes the appearance from new physics contributions to the effective couplings and observables considered here involving the photon and the gluon. As we have seen this can be understood on rather general grounds but it is still nice to see it explicitly.

We also observe at once that many complex phases appear (or disappear) in the coefficients of the effective operators after the passage to the physical basis. Even if the ma-

trices $M_{L,R}$ were real (and thus the effective operators themselves preserved CP) phases do appear after the diagonalization, both due to the appearance of the usual CKM matrix in those effective operators involving left-handed fields, but also because the diagonalization matrices appear explicitly, both for left and right-handed operators. Furthermore the effective operators couplings are redefined by matrices which are not unitary in general. It is conceivable that this might enhance slightly the CP violation induced by the effective operators, for instance very large custodially breaking contributions in the new physics (provided that these evade the rather stringent bounds coming from the ρ parameter [11]) would give rather different values to the matrices X_{Ru} and X_{Rd} , yielding eigenvalues smaller than one in one of the two. These might enhance CP violation in the right-handed sector.

In the standard model there is a link between the existence of three families and the presence of CP violation. This disappears completely, both in the left and right-handed sectors, once additional operators are included. The new CP -violating contributions need not, in fact, be suppressed by the product of all the mass differences, as it happens in the standard model. This is obviously so if the physics responsible for the effective operators in the weak basis is CP -violating, but even if it turns out that the new physics is such that the effective operators do not violate CP in the weak basis, both the effective left and right-handed couplings contain many independent phases as pointed out in Sec. II. Indeed from Eqs. (24)–(27) we see that we can have up to 9 independent phases in the left sector (1 in K and the other 8 in the N 's, the latter not observable in the standard model) and from Eqs. (24) and (32) we see we can have up to 18 independent phases in the right sector which were not observable in the standard model. (See [12] for some work on right-handed phases and mixing matrices.) Obviously if the matrices M are allowed to be complex more phases are available.

How can we check for the presence of all this wealth of new phases? In the left-handed sector the analysis is usually done in terms of the unitarity triangle. Clearly the unitarity triangle as such is gone once the additional $d=4$ operators are included. To see this we need only to examine Eq. (42). The total charged current vertex will be proportional to

$$\mathcal{U} = K + GK, \quad (50)$$

where G is a combination of the N matrices. Since G is not anti-Hermitian, $\mathcal{U}$ is not unitary in a perturbative sense. This of course is what happens when the contribution from the new physics is considered, but it is clear that this will happen in the standard model too when radiative corrections are included, since radiative corrections give very specific, but non-zero, values for the effective couplings which also lead to violations of unitarity.

However, these deviations of unitarity due to radiative corrections shall be small. We expect contributions of order $g^2/16\pi^2$ from the gauge sector and of order $(y^2/16\pi^2)\log M_H^2/M_W^2$ from the scalar sector to the couplings; at most of order a few times 10^{-3} . This is almost certainly

invisible in the ongoing generation of experiments trying to test the CP -violating sector of the standard model. Deviations from the tree-level predictions, expressed through the coefficients of the effective Lagrangian and their effective coupling counterpart will be measurable at present only if they are sizably larger than the radiative corrections themselves. It is not so easy, however, to build models where this is so. We refer the reader to Sec. VI for a few more comments on this. We would also like to draw the reader's attention to [13] and references therein.

V. RADIATIVE CORRECTIONS AND RENORMALIZATION

As we mentioned in the previous section, the effective couplings presented in Eq. (42) for the charged current vertices are not the complete story because CKM and wave-function renormalization gives a non-trivial contribution there. In this section we shall consider the contribution to the observables due to wave-function renormalization and the renormalization of the CKM matrix elements. The issue, we shall see, is far from trivial.

When we calculate cross sections in perturbation theory we have to take into account the residues of the external leg propagators. The meaning of these residues is clear when we do not have mixing. In this case, if we work in the on-shell scheme, we can attempt to absorb these residues in the wave function renormalization constants and forget about them. However the Ward identities force us to set up relations between the renormalization constants that invalidate the naive on-shell scheme [14]. The issue is resolved in the following way: Take whatever renormalization scheme that respects Ward identities and use the corresponding renormalization constants everywhere in except for the external legs contributions. For the latter we just have to impose the mass pole and unit residue conditions. This recipe is equivalent to use the Ward identities-complying renormalization constants everywhere and afterwards perform a finite renormalization of the external fields in order to assure mass pole and residue one for the propagators. This is the commonly used prescription in the context of the popular and convenient on-shell scheme [14] and, in the context of effective theories was used in [15] and in [5].

Now let us now turn to the case where we have mixing. This was studied some time ago by Aoki *et al.* [16] and a on-shell scheme was proposed. Unfortunately the issue is not settled. We have studied the problem with some detail anew since, as already mentioned, the contribution from wave-function renormalization is important in the present case. We have found out that the set of conditions imposed by Aoki *et al.* over-determine the renormalization constants and is in fact incompatible with the analytic structure of the theory. Moreover, even if this problem is ignored, it was found some time ago [17] that the proposal conflicts with the Becchi-Rouet-Stora-Tyutin (BRST) symmetry of the theory. Therefore, now we will analyze the renormalization issue with some detail and then we shall propose a couple of schemes which are free of the over-determination problem. Once we have obtained those schemes we will show how they must be

used in order to avoid conflict with Ward identities.

The renormalized fermionic propagator is given by

$$\begin{aligned}
 S(p) &= \frac{i}{\not{p} - m - \hat{\Sigma}(p)} \\
 &= i(\not{p} - m - \hat{\Sigma}(p))^{-1} \\
 &= i[(1 - \hat{\Sigma}(p)(\not{p} - m)^{-1})(\not{p} - m)]^{-1} \\
 &= i(\not{p} - m)^{-1}(1 - [-i\hat{\Sigma}(p)]i(\not{p} - m)^{-1})^{-1} \\
 &= i(\not{p} - m)^{-1} + i(\not{p} - m)^{-1}[-i\hat{\Sigma}(p)]i(\not{p} - m)^{-1} + \dots,
 \end{aligned} \tag{51}$$

where, since we have mixing, the renormalized self-energy $\hat{\Sigma}(p)$ have family indices. Unless explicitly said otherwise, all expressions are valid both for up and down type fermions. We will indicate the weak indices u or d only when necessary. From Poincaré invariance we can write

$$\hat{\Sigma}_{ij}(p) = \not{p}(\hat{\Sigma}_{ij}^{\gamma R}(p^2)R + \hat{\Sigma}_{ij}^{\gamma L}(p^2)L) + \hat{\Sigma}_{ij}^R(p^2)R + \hat{\Sigma}_{ij}^L(p^2)L, \tag{52}$$

where L and R are left and right projectors respectively, so

$$S_{ij}^{-1}(p) = -i(\not{p} - m - \hat{\Sigma}(p))_{ij} = -i(\not{p} - m_i)\delta_{ij} + i\hat{\Sigma}_{ij}(p). \tag{53}$$

The on-shell conditions given by Aoki *et al.* are

$$S_{ij}^{-1}(p_j)u_j^s(p_j) = 0, \tag{54}$$

$$\bar{u}_i^s(p_i)S_{ij}^{-1}(p_i) = 0, \tag{55}$$

$$i(\not{p}_i - m_i)^{-1}S_{ii}^{-1}(p_i)u_i^s(p_i) = u_i^s(p_i), \tag{56}$$

$$\bar{u}_i^s(p_i)S_{ii}^{-1}(p_i)i(\not{p}_i - m_i)^{-1} = \bar{u}_i^s(p_i), \tag{57}$$

where we do not sum over repeated indices and where $p_i^2 \rightarrow m_i^2$ (on-shell). With u_j^s we indicate the Dirac spinor satisfying the on-shell condition

$$(\not{p}_i - m_i)u_i^s(p_i) = 0. \tag{58}$$

From Eqs. (53) and (54) we obtain

$$\begin{aligned}
 &([\hat{\Sigma}_{ij}^{\gamma L}(m_j^2)m_j + \hat{\Sigma}_{ij}^R(m_j^2)]R \\
 &+ [\hat{\Sigma}_{ij}^{\gamma R}(m_j^2)m_j + \hat{\Sigma}_{ij}^L(m_j^2)]L)u_j^s(p_j) = 0,
 \end{aligned} \tag{59}$$

and from there

$$\hat{\Sigma}_{ij}^{\gamma L}(m_j^2)m_j + \hat{\Sigma}_{ij}^R(m_j^2) = 0, \tag{60}$$

$$\hat{\Sigma}_{ij}^{\gamma R}(m_j^2)m_j + \hat{\Sigma}_{ij}^L(m_j^2) = 0.$$

Analogously from Eqs. (53) and (55) we obtain

$$\begin{aligned}
 &\bar{u}_i^s(p_i)(m_i\hat{\Sigma}_{ij}^{\gamma R}(m_i^2)R + m_i\hat{\Sigma}_{ij}^{\gamma L}(m_i^2)L + \hat{\Sigma}_{ij}^R(m_i^2)R \\
 &+ \hat{\Sigma}_{ij}^L(m_i^2)L) = 0,
 \end{aligned} \tag{61}$$

and from there

$$m_i\hat{\Sigma}_{ij}^{\gamma R}(m_i^2) + \hat{\Sigma}_{ij}^R(m_i^2) = 0, \tag{62}$$

$$m_i\hat{\Sigma}_{ij}^{\gamma L}(m_i^2) + \hat{\Sigma}_{ij}^L(m_i^2) = 0.$$

From Eqs. (53) and (56) we obtain

$$\begin{aligned}
 &\hat{\Sigma}_{ii}^{\gamma R}(m_i^2) + m_i^2(\hat{\Sigma}_{ii}^{\gamma R'}(m_i^2) + \hat{\Sigma}_{ii}^{\gamma L'}(m_i^2)) + m_i(\hat{\Sigma}_{ii}^{R'}(m_i^2) + \hat{\Sigma}_{ii}^{L'}(m_i^2)) = 0, \\
 &\hat{\Sigma}_{ii}^{\gamma L}(m_i^2) + m_i^2(\hat{\Sigma}_{ii}^{\gamma L'}(m_i^2) + \hat{\Sigma}_{ii}^{\gamma R'}(m_i^2)) + m_i(\hat{\Sigma}_{ii}^{L'}(m_i^2) + \hat{\Sigma}_{ii}^{R'}(m_i^2)) = 0,
 \end{aligned} \tag{63}$$

and finally from Eqs. (53) and (57) we obtain again the same equations that we have derived from the condition (56). So we can write the whole set of Aoki *et al.* renormalization conditions as

$$\begin{aligned}
 0 &= \hat{\Sigma}_{ij}^{\gamma L}(m_j^2)m_j + \hat{\Sigma}_{ij}^R(m_j^2), \\
 0 &= m_i\hat{\Sigma}_{ij}^{\gamma R}(m_i^2) + \hat{\Sigma}_{ij}^R(m_i^2), \\
 0 &= \hat{\Sigma}_{ii}^{\gamma R}(m_i^2) + m_i^2(\hat{\Sigma}_{ii}^{\gamma R'}(m_i^2) + \hat{\Sigma}_{ii}^{\gamma L'}(m_i^2)) \\
 &\quad + m_i(\hat{\Sigma}_{ii}^{R'}(m_i^2) + \hat{\Sigma}_{ii}^{L'}(m_i^2)),
 \end{aligned} \tag{64}$$

as well as those obtained by the exchange $R \leftrightarrow L$.

With the help of the mass counterterm and the left and right wave-function renormalization constants the renormalized self energy $\hat{\Sigma}_{ij}$ can be written as

$$\begin{aligned}
 \hat{\Sigma}_{ij} &= \Sigma_{ij} - \frac{1}{2}\not{p}L(\delta Z_{ij}^{L\dagger} + \delta Z_{ij}^L) - \frac{1}{2}\not{p}R(\delta Z_{ij}^{R\dagger} + \delta Z_{ij}^R) \\
 &\quad + \frac{1}{2}R(\delta Z_{ij}^{L\dagger}m_j + m_i\delta Z_{ij}^R) + \frac{1}{2}L(\delta Z_{ij}^{R\dagger}m_j + m_i\delta Z_{ij}^L) \\
 &\quad + \delta_{ij}\delta m_i,
 \end{aligned} \tag{65}$$

where Σ_{ij} is the bare self-energy. Using Eqs. (52), (64), and (65) we can obtain the following relations among bare self-energies

$$(\Sigma_{ij}^L(m_j^2) - \Sigma_{ij}^{L\dagger}(m_j^2))m_j + (\Sigma_{ij}^R(m_j^2) - \Sigma_{ij}^{L\dagger}(m_j^2)) = 0, \quad (66)$$

and a similar relation exchanging $R \leftrightarrow L$. But we know that this relations are not satisfied because self-energies are not Hermitian due, e.g., to the branch cut generated by the loop of massless virtual fotons. The appearance of this type of (false) relations is due to the over-determination of conditions (54)–(57).

There are several ways to solve this over-determination, here we will present the ones that we believe are more physical.

A. “Incoming fermion” scheme

To avoid over-determination we will define the following renormalization conditions. We will keep for $i \neq j$ the Aoki *et al.* renormalization condition (54) namely

$$S_{ij}^{-1}(p)u_j^s(p) = 0 \quad i \neq j, \quad p^2 \rightarrow m_j^2, \quad (67)$$

which physically means that we have no mixing on shell of the incoming fermions and in terms of self energies amounts to

$$0 = \hat{\Sigma}_{ij}^L(m_j^2)m_j + \hat{\Sigma}_{ij}^R(m_j^2), \quad i \neq j, \quad (68)$$

and a similar condition exchanging $R \leftrightarrow L$. For $i = j$ we only impose this condition over the real part of the inverse propagator

$$\text{Re}(iS^{-1})_{ii}(p)u_i^s(p) = 0 \quad p^2 \rightarrow m_i^2, \quad (69)$$

the restriction to the real part is necessary because fermions need not be stable particles (in fact they are not in general) so an appropriate condition for the mass pole is Eq. (69), which in terms of self energies amounts to

$$0 = (\hat{\Sigma}_{ii}^R(m_i^2) + \hat{\Sigma}_{ii}^{R\dagger}(m_i^2))m_i + \hat{\Sigma}_{ii}^L(m_i^2) + \hat{\Sigma}_{ii}^{L\dagger}(m_i^2), \quad (70)$$

and a similar condition exchanging $R \leftrightarrow L$. We also add the unit residue condition

$$(\not{p} - m_i)^{-1} \text{Re}(iS^{-1})_{ii}(p)u_i^s(p) = u_i^s(p), \quad p^2 \rightarrow m_i^2, \quad (71)$$

which can be shown to be equivalent to

$$\overline{u_i^s}(p) \text{Re}(iS^{-1})_{ii}(p)(\not{p} - m_i)^{-1} = \overline{u_i^s}(p), \quad p^2 \rightarrow m_i^2. \quad (72)$$

The diagonal antihermitian parts of the bare self-energy are finite, so it can be shown that in order to keep the renormalized ones finite we only need to impose

$$\delta Z_{ii}^L - \delta Z_{ii}^{L\dagger} = \delta Z_{ii}^R - \delta Z_{ii}^{R\dagger} + \text{const.} \quad (73)$$

In the on-shell scheme without mixing $\delta Z_{ii}^L - \delta Z_{ii}^{L\dagger} = \delta Z_{ii}^R - \delta Z_{ii}^{R\dagger} = 0$ is tacitly assumed. However due to the rephasing freedom only condition (73) is necessary to absorb all the divergencies. Here, for simplicity reasons, we also take

$$\delta Z_{ii}^L - \delta Z_{ii}^{L\dagger} = \delta Z_{ii}^R - \delta Z_{ii}^{R\dagger} = 0. \quad (74)$$

Note that apart from taking the real part in Eqs. (69), (71), (72) we have also omitted Aoki *et al.* condition (55) to avoid over-determination. We can expect that another set of consistent condition that include condition (55) (for $i \neq j$, and taking the real part in the diagonal case) can be given, and actually this is the case.

Performing the calculations in the incoming fermion scheme we obtain the following set of wave-function renormalization constants:

$$\begin{aligned} \delta Z_{ij}^{L\dagger} + \delta Z_{ij}^L = & \frac{2}{m_j^2 - m_i^2} \{ \Sigma_{ij}^L(m_j^2)m_j^2 - \Sigma_{ij}^{L\dagger}(m_i^2)m_i^2 \\ & + \Sigma_{ij}^R(m_j^2)m_i m_j - \Sigma_{ij}^{R\dagger}(m_i^2)m_j m_i \\ & + \Sigma_{ij}^R(m_j^2)m_j - \Sigma_{ij}^{R\dagger}(m_i^2)m_i + \Sigma_{ij}^L(m_j^2)m_i \\ & - \Sigma_{ij}^{L\dagger}(m_i^2)m_j \} \quad (i \neq j), \end{aligned} \quad (75)$$

$$\begin{aligned} \delta Z_{ij}^L - \delta Z_{ij}^{L\dagger} = & \frac{2}{m_j^2 - m_i^2} \{ \Sigma_{ij}^L(m_j^2)m_j^2 + \Sigma_{ij}^{L\dagger}(m_i^2)m_i^2 \\ & + \Sigma_{ij}^R(m_j^2)m_j m_i + \Sigma_{ij}^{R\dagger}(m_i^2)m_j m_i \\ & + \Sigma_{ij}^L(m_j^2)m_i + \Sigma_{ij}^{L\dagger}(m_i^2)m_j + \Sigma_{ij}^R(m_j^2)m_j \\ & + \Sigma_{ij}^{R\dagger}(m_i^2)m_i \} \quad (i \neq j), \end{aligned} \quad (76)$$

$$\begin{aligned} \delta Z_{ii}^{R\dagger} + \delta Z_{ii}^R = & m_i^2 (\Sigma_{ii}^{R'}(m_i^2) + \Sigma_{ii}^{R'\dagger}(m_i^2) + \Sigma_{ii}^{L'}(m_i^2) \\ & + \Sigma_{ii}^{L'\dagger}(m_i^2)) + m_i (\Sigma_{ii}^{R'}(m_i^2) + \Sigma_{ii}^{R'\dagger}(m_i^2) \\ & + \Sigma_{ii}^{L'}(m_i^2) + \Sigma_{ii}^{L'\dagger}(m_i^2)) + \Sigma_{ii}^R(m_i^2) \\ & + \Sigma_{ii}^{R\dagger}(m_i^2), \end{aligned} \quad (77)$$

and, as usual, similar conditions obtained after the exchange $R \leftrightarrow L$.

We also have we also have

$$\begin{aligned} \delta m_i = & -\frac{1}{4} \{ (\Sigma_{ii}^L(m_i^2) + \Sigma_{ii}^{L\dagger}(m_i^2) + \Sigma_{ii}^R(m_i^2) \\ & + \Sigma_{ii}^{R\dagger}(m_i^2))m_i + \Sigma_{ii}^R(m_i^2) + \Sigma_{ii}^{R\dagger}(m_i^2) + \Sigma_{ii}^L(m_i^2) \\ & + \Sigma_{ii}^{L\dagger}(m_i^2) \}. \end{aligned} \quad (78)$$

Here it is worth noting that even though this scheme has less conditions than the Aoki *et al.* set we still obtain restrictions over bare self energies, namely

$$\Sigma_{ii}^L(m_i^2) + \Sigma_{ii}^{L\dagger}(m_i^2) = \Sigma_{ii}^R(m_i^2) + \Sigma_{ii}^{R\dagger}(m_i^2), \quad (79)$$

but in this case it can be seen by direct calculation to one loop that this relation holds.

B. “Outcoming fermion” scheme

Another possibility is to define an on-shell scheme by the following set of conditions. We impose

$$\bar{u}_i^s(p) S_{ij}^{-1}(p) = 0 \quad (i \neq j, \quad p^2 \rightarrow m_i^2), \quad (80)$$

which physically means that we have no mixing on shell of the outcoming fermions and in terms of self energies amounts to

$$0 = m_i \hat{\Sigma}_{ij}^R(m_i^2) + \hat{\Sigma}_{ij}^R(m_i^2), \quad (81)$$

plus the $R \leftrightarrow L$ condition.

For $i=j$ we again impose this condition only over $\text{Re}(iS^{-1})$ that is

$$\bar{u}_i^s(p) \text{Re}(iS^{-1})_{ii}(p) = 0, \quad p^2 \rightarrow m_i^2, \quad (82)$$

which in terms of self energies amounts to

$$0 = (\hat{\Sigma}_{ii}^R(m_i^2) + \hat{\Sigma}_{ii}^{R\dagger}(m_i^2))m_i + \hat{\Sigma}_{ii}^R(m_i^2) + \hat{\Sigma}_{ii}^{R\dagger}(m_i^2), \quad (83)$$

and, as customary, the exchanged $R \leftrightarrow L$ condition. The unit residue conditions are the same as in the incoming fermion scheme.

Performing the calculations in the outcoming fermion scheme we obtain the following set of wave-function renormalization constants:

$$\begin{aligned} \delta Z_{ij}^L + \delta Z_{ij}^{L\dagger} &= \frac{2}{m_i^2 - m_j^2} \{ \Sigma_{ij}^L(m_i^2)m_i^2 - \Sigma_{ij}^{L\dagger}(m_j^2)m_j^2 \\ &+ \Sigma_{ij}^R(m_i^2)m_i m_j - \Sigma_{ij}^{R\dagger}(m_j^2)m_j m_i \\ &+ \Sigma_{ij}^R(m_i^2)m_j - \Sigma_{ij}^{R\dagger}(m_j^2)m_i + \Sigma_{ij}^L(m_i^2)m_i \\ &- \Sigma_{ij}^{L\dagger}(m_j^2)m_j \} \quad (i \neq j), \end{aligned} \quad (84)$$

$$\begin{aligned} \delta Z_{ij}^L - \delta Z_{ij}^{L\dagger} &= \frac{2}{m_j^2 - m_i^2} \{ m_i \Sigma_{ij}^L(m_i^2) + m_j \Sigma_{ij}^{L\dagger}(m_j^2) \\ &+ m_j \Sigma_{ij}^R(m_i^2) + m_i \Sigma_{ij}^{R\dagger}(m_j^2) \\ &+ m_j m_i \Sigma_{ij}^R(m_i^2) + m_i m_j \Sigma_{ij}^{R\dagger}(m_j^2) \\ &+ m_i^2 \Sigma_{ij}^L(m_i^2) + m_j^2 \Sigma_{ij}^{L\dagger}(m_j^2) \} \quad (i \neq j), \end{aligned} \quad (85)$$

$$\begin{aligned} \delta Z_{ii}^{R\dagger} + \delta Z_{ii}^R &= m_i^2 (\Sigma_{ii}^{R'}(m_i^2) + \Sigma_{ii}^{R'\dagger}(m_i^2) + \Sigma_{ii}^{L'}(m_i^2) \\ &+ \Sigma_{ii}^{L'\dagger}(m_i^2)) + m_i (\Sigma_{ii}^{R'}(m_i^2) + \Sigma_{ii}^{R'\dagger}(m_i^2) \\ &+ \Sigma_{ii}^{L'}(m_i^2) + \Sigma_{ii}^{L'\dagger}(m_i^2)) + \Sigma_{ii}^R(m_i^2) \\ &+ \Sigma_{ii}^{R\dagger}(m_i^2), \end{aligned} \quad (86)$$

and, in addition, those obtained after the replacement $R \leftrightarrow L$. The mass counterterm is identical to the one obtained in the incoming fermion scheme.

Here again we obtain the relation (79). Note that diagonal counterterms coincide in both schemes while this is not the case for off-diagonal ones and of course when there is no mixing the usual renormalization constants are reproduced.

So far we have presented the above schemes without specifying weak indices u or d . In the next subsections we will see that the above schemes can be imposed alternatively on up or down type fermions but not *on both at the same time*. The reason is that gauge symmetry impose certain relations between renormalization constants that are not fulfilled in the former case.

C. The role of Ward identities

Let us obtain the Ward identities that relate renormalization constants in the physical base. The non-physical base belongs to an irreducible representation of $SU_L(2)$ (weak doublet) and we want the renormalization group to respect this representation, that is

$$u_L^0 = Z^{L(1/2)} u_L, \quad (87)$$

$$d_L^0 = Z^{L(1/2)} d_L,$$

where the wave function renormalization $Z^{L(1/2)}$ is the same for the two components of the weak doublet. The non-physical basis is related to the physical one via a bi-unitary transformation given by

$$u_L^0 = V_{Lu}^0 u_L^0, \quad u_L = V_{Lu} u_L, \quad (88)$$

$$d_L^0 = V_{Ld}^0 d_L^0, \quad d_L = V_{Ld} d_L,$$

so we obtain

$$u_L^0 = V_{Lu}^{0\dagger} Z^{L(1/2)} V_{Lu} u_L \equiv Z^{uL(1/2)} u_L, \quad (89)$$

$$d_L^0 = V_{Ld}^{0\dagger} Z^{L(1/2)} V_{Ld} d_L \equiv Z^{dL(1/2)} d_L,$$

where we have defined the wave function renormalization for the up and down flavors in the physical basis as $Z^{uL(1/2)} = V_{Lu}^{0\dagger} Z^{L(1/2)} V_{Lu}$ and $Z^{dL(1/2)} = V_{Ld}^{0\dagger} Z^{L(1/2)} V_{Ld}$ respectively. From Eqs. (89) we immediately obtain [18]

$$K^0 = V_{Lu}^{0\dagger} V_{Ld}^0 = Z^{uL(1/2)} V_{Lu}^\dagger V_{Ld} Z^{dL-(1/2)} = Z^{uL(1/2)} K Z^{dL-(1/2)}, \quad (90)$$

and

$$\begin{aligned} Z^{uL\dagger(1/2)} Z^{uL(1/2)} &= V_{Lu}^\dagger Z^{L\dagger(1/2)} Z^{L(1/2)} V_{Lu} \\ &= V_{Lu}^\dagger V_{Ld} Z^{dL\dagger(1/2)} Z^{dL(1/2)} V_{Ld}^\dagger V_{Lu} \\ &= K Z^{dL\dagger(1/2)} Z^{dL(1/2)} K^\dagger. \end{aligned} \quad (91)$$

If we define the CKM renormalization constant as $K^0 = K + \delta K$ we can rewrite Eqs. (90) and (91) in perturbation theory as

$$\delta K = \frac{1}{2}(\delta Z^{uL}K - K\delta Z^{dL}), \quad (92)$$

$$\delta Z^{uL\dagger} + \delta Z^{uL} = K(\delta Z^{dL\dagger} + \delta Z^{dL})K^\dagger. \quad (93)$$

Equations (92) and (93) relating renormalization constants in the physical base are consequence of $SU_L(2)$ gauge invariance and must be satisfied by any renormalization scheme.

Now we can see that a simple solution to obtain all renormalization constants respecting Ward identities is to impose one of the presented on-shell schemes for the down (up) fermions and then use Eq. (93) to obtain the left Hermitian part of the wave function for the up (down) fermions. For the anti-Hermitian and right Hermitian parts of the up (down) fermions we can use the same expressions used for the down (up), but with the $u \leftrightarrow d$ replacement. This procedure leads to a finite set of Green functions and it is obviously compliant with the Ward identities. However, this procedure alone does not lead to up and down propagators with the desired properties listed in one of the two on-shell schemes. Thus for external legs the above renormalization prescription must be supplemented with an additional finite renormalization, ensuring the compliance with the incoming or outgoing schemes (depending whether the particle is in the in or out state). We will illustrate this point in the next section where we calculate the contribution to the renormalization of the CKM matrix given by Eq. (92) and the wave function renormalization which in the effective Lagrangian comes in both cases solely from $\mathcal{L}_L^4$.

D. Contribution of $\mathcal{L}_L^4$ to wave-function renormalization

The operator $\mathcal{L}_L^4$ is the only one contributing to self-energies and, hence, to the wave-function renormalization constants. It also gives a contribution (among others) to the neutral current vertices which [see Eq. (39)], when compared to the tree level standard model contribution, is proportional to

$$[(N^4 + N^{4\dagger})\tau^u - (K^\dagger(N^4 + N^{4\dagger})K)\tau^d]L. \quad (94)$$

The contribution from $\mathcal{L}_L^4$ to the bare self-energies is

$$\begin{aligned} \Sigma^{R(u,d)} &= \Sigma^{L(u,d)} = 0, \\ \Sigma^{\gamma Ru} &= \Sigma^{\gamma Rd} = 0, \\ \Sigma^{\gamma Ld} &= K^\dagger(N^4 + N^{4\dagger})K, \\ \Sigma^{\gamma Lu} &= -(N^4 + N^{4\dagger}), \end{aligned} \quad (95)$$

hence using either the incoming or outgoing on-shell renormalization conditions we obtain (both give identical results in the present case, but note that this is not true in general)

$$\frac{1}{2}(\delta Z_{ij}^{dL} + \delta Z_{ij}^{dL\dagger}) = (K^\dagger(N^4 + N^{4\dagger})K)_{ij}, \quad (96)$$

$$\frac{1}{2}(\delta Z_{ij}^{dR} + \delta Z_{ij}^{dR\dagger}) = 0, \quad (97)$$

$$\frac{1}{2}(\delta Z_{ij}^{dL} - \delta Z_{ij}^{dL\dagger}) = \frac{m_j^{d2} + m_i^{d2}}{m_j^{d2} - m_i^{d2}}(K^\dagger(N^4 + N^{4\dagger})K)_{ij} \quad (i \neq j), \quad (98)$$

$$\frac{1}{2}(\delta Z_{ij}^{dR} - \delta Z_{ij}^{dR\dagger}) = \frac{2m_i^d m_j^d}{m_j^{d2} - m_i^{d2}}(K^\dagger(N^4 + N^{4\dagger})K)_{ij} \quad (i \neq j), \quad (99)$$

$$\frac{1}{2}(\delta Z_{ii}^{dL} - \delta Z_{ii}^{dL\dagger}) = \frac{1}{2}(\delta Z_{ii}^{dR} - \delta Z_{ii}^{dR\dagger}) = 0. \quad (100)$$

Had we have used the same conditions for the up fermions we would have obtained

$$\frac{1}{2}(\delta Z_{ij}^{uL} + \delta Z_{ij}^{uL\dagger}) = -(N^4 + N^{4\dagger})_{ij}, \quad (101)$$

$$\frac{1}{2}(\delta Z_{ij}^{uR} + \delta Z_{ij}^{uR\dagger}) = 0, \quad (102)$$

$$\frac{1}{2}(\delta Z_{ij}^{uL} - \delta Z_{ij}^{uL\dagger}) = -\frac{m_j^{u2} + m_i^{u2}}{m_j^{u2} - m_i^{u2}}(N^4 + N^{4\dagger})_{ij} \quad (i \neq j), \quad (103)$$

$$\frac{1}{2}(\delta Z_{ij}^{uR} - \delta Z_{ij}^{uR\dagger}) = -\frac{2m_i^u m_j^u}{m_j^{u2} - m_i^{u2}}(N^4 + N^{4\dagger})_{ij} \quad (i \neq j), \quad (104)$$

$$\frac{1}{2}(\delta Z_{ii}^{uL} - \delta Z_{ii}^{uL\dagger}) = \frac{1}{2}(\delta Z_{ii}^{uR} - \delta Z_{ii}^{uR\dagger}) = 0, \quad (105)$$

note that Eqs. (96) and (101) are indeed incompatible with the Ward identity (93) as expected. A solution to this incompatibility is simply to take one of the two sets as valid for one of the fermions (or even none of them; for example we can use the minimal scheme), use the Ward identity to determine the left Hermitian part of the renormalization of the other fermion, while keeping the anti-Hermitian and right Hermitian parts from the original prescription. The renormalization of the CKM matrix is then fixed by Eq. (92). Then we proceed to renormalize the external fermions with additional finite renormalization constants $\hat{Z}^{uL(1/2)}$ and $\hat{Z}^{dL(1/2)}$ with $\hat{Z}^{uL(1/2)}Z^{uL(1/2)}$ and $\hat{Z}^{dL(1/2)}Z^{dL(1/2)}$ satisfying the incoming or outgoing schemes, as appropriate. For instance a consistent scheme in the present case would be to retain Eqs. (96)–(100), and then Eqs. (102)–(105). Then replace Eq. (101) by (93), which implies

$$\frac{1}{2}(\delta Z_{ij}^{uL} + \delta Z_{ij}^{uL\dagger}) = (N^4 + N^{4\dagger})_{ij}. \quad (106)$$

Note the sign difference with respect to Eq. (101).

The above one is a Ward identity-compliant set of wave function renormalization constants. From them, it is immediate to read the way the CKM matrix renormalizes. As for the additional (finite, if radiative corrections were included) renormalization, in the present case this amounts to

$$\delta\hat{Z}_{ij}^{dL}=0,$$

$$\frac{1}{2}(\delta\hat{Z}_{ij}^{uL} + \delta\hat{Z}_{ij}^{uL\dagger}) = -2(N^4 + N^{4\dagger})_{ij}, \quad (107)$$

$$\frac{1}{2}(\delta\hat{Z}_{ij}^{uL} - \delta\hat{Z}_{ij}^{uL\dagger}) = 0,$$

but the whole procedure is (for the external legs) equivalent to use directly Eqs. (96)–(105) in the first place.

The bare kinetic term in the physical base in the standard model is given by

$$\mathcal{L}_{kin} = i\bar{f}\gamma^\mu \left\{ \partial_\mu + i\frac{e}{s_W} \left(K\frac{\tau^-}{2}W_\mu^+ + K^\dagger\frac{\tau^+}{2}W_\mu^- \right) L + ieQA_\mu + i\frac{e}{c_W s_W} \left[\left(\frac{\tau^3}{2} - Qs_W^2 \right) L - Qs_W^2 R \right] Z_\mu + ig_s \frac{\lambda}{2} \cdot G_\mu \right\} f. \quad (108)$$

To calculate the tree level contribution of $\mathcal{L}_L^4$ via this term after renormalization we write it as

$$\begin{aligned} \mathcal{L}_{kin} \rightarrow i\bar{f}\gamma^\mu & \left[(\hat{Z}^{uL\dagger(1/2)} Z^{uL(1/2)} L + Z^{uR\dagger(1/2)} R) \tau^\mu + (\hat{Z}^{dL\dagger(1/2)} Z^{dL(1/2)} L + Z^{dR\dagger(1/2)} R) \tau^d \right] \left\{ \partial_\mu + i\frac{e}{s_W} \left(Z^{uL(1/2)} K Z^{dL(-1/2)} \frac{\tau^-}{2} W_\mu^+ \right. \right. \\ & \left. \left. + Z^{dL(-1/2)} K^\dagger Z^{uL(1/2)} \frac{\tau^+}{2} W_\mu^- \right) L + ieQA_\mu + i\frac{e}{c_W s_W} \left[\left(\frac{\tau^3}{2} - Qs_W^2 \right) L - Qs_W^2 R \right] Z_\mu + ig_s \frac{\lambda}{2} \cdot G_\mu \right\} \\ & \times [(Z^{uL(1/2)} \hat{Z}^{uL(1/2)} L + Z^{uR(1/2)} R) \tau^\mu + (Z^{dL(1/2)} \hat{Z}^{dL(1/2)} L + Z^{dR(1/2)} R) \tau^d] f, \end{aligned} \quad (109)$$

where we have introduced the additional finite renormalization constants $\hat{Z}^{uL(1/2)}$ and $\hat{Z}^{dL(1/2)}$ necessary to avoid mixing and maintain residue 1 in the propagators. We have also renormalized K according to the Ward identity (90). With the renormalization constants taken into account we observe that the total contribution of $\mathcal{L}_L^4$ to the neutral current vertices vanishes. This is a very non-trivial check of the whole procedure. Of course nothing prevents the appearance of N^4 at higher orders when one, for instance, performs loops with the effective operators. But this is a purely academic question at this point.

Finally let us see what happens to the charged current vertices. The total contribution of $\mathcal{L}_{kin}$ and $\mathcal{L}_L^4$ including renormalization constants to the charged vertex is

$$\begin{aligned} & \left(I + \frac{\delta\hat{Z}^{uL} + \delta\hat{Z}^{uL\dagger}}{4} - \frac{\delta\hat{Z}^{uL} - \delta\hat{Z}^{uL\dagger}}{4} \right) \left(I + \frac{\delta Z^{uL} + \delta Z^{uL\dagger}}{2} + (N^4 - N^{4\dagger}) \right) K \left(I + \frac{\delta\hat{Z}^{dL} + \delta\hat{Z}^{dL\dagger}}{4} + \frac{\delta\hat{Z}^{dL} - \delta\hat{Z}^{dL\dagger}}{4} \right) \\ & = \left(I + \frac{\delta\hat{Z}^{uL} + \delta\hat{Z}^{uL\dagger}}{4} - \frac{\delta\hat{Z}^{uL} - \delta\hat{Z}^{uL\dagger}}{4} + \frac{\delta Z^{uL} + \delta Z^{uL\dagger}}{2} + (N^4 - N^{4\dagger}) \right) K + K \left(\frac{\delta\hat{Z}^{dL} + \delta\hat{Z}^{dL\dagger}}{4} + \frac{\delta\hat{Z}^{dL} - \delta\hat{Z}^{dL\dagger}}{4} \right) \\ & = \left(I + (N^4 - N^{4\dagger}) - \frac{\delta\hat{Z}^{uL} - \delta\hat{Z}^{uL\dagger}}{4} \right) K + K \left(\frac{\delta\hat{Z}^{dL} - \delta\hat{Z}^{dL\dagger}}{4} \right) \\ & = K + (N^4 - N^{4\dagger})K, \end{aligned} \quad (110)$$

where we have used the Ward identity (93) along with Eqs. (96)–(100), Eqs. (102)–(105) and Eq. (107). We observe that the total contribution of $\mathcal{L}_{kin} + \mathcal{L}_L^4$ is in fact equal to the contribution of $\mathcal{L}_L^4$ alone. The contributions coming from the wave function and CKM renormalizations cancel out at tree level. Another point to note is that this particular contribution preserves the perturbative unitarity of K , in accordance with the equations-of-motion argument. This completes the theoretical analysis of the CKM and wave-function renormalization.

VI. SOME EXAMPLES

Let us now try to get a feeling about the order of magnitude of the coefficients of the effective Lagrangian. We shall consider two examples: the effective theory induced by the integration of a heavy doublet and the standard model itself in the limit of a heavy Higgs boson.

In the heavy doublet case we shall make use of some recent work by Del Aguila and co-workers [19]. These authors have recently analyzed the effect of integrating out heavy matter fields in different representations. For illustra-

tion purposes we shall only consider the doublet case here. As emphasized in [19] while additional chiral doublets are surely excluded by the LEP data, vector multiplets are not.

Let us assume that the standard model is extended with a doublet of heavy fermions Q of mass M , with vector coupling to the gauge field. For the time being we shall assume a light Higgs boson. In addition there will be an extended Higgs-Yukawa term of the form

$$\lambda_j^{(u)} \bar{Q} \tilde{\phi} R u_j + \lambda_j^{(d)} \bar{Q} \phi R d_j, \quad (111)$$

where

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ v + h + i\varphi_3 \end{pmatrix}, \quad \tilde{\phi} = i\tau^2 \phi^*, \quad f = \begin{pmatrix} u \\ d \end{pmatrix}. \quad (112)$$

The heavy doublet can be exactly integrated. This procedure is described in detail in [19]. After this operation we generate the following effective couplings (all of them corresponding to operators of dimension six)

$$\begin{aligned} & i\phi^\dagger D_\mu \phi \bar{f} \alpha_{\phi q}^{(1)} \gamma^\mu L f + \text{H.c.}, \\ & i\phi^\dagger \tau^j D_\mu \phi \bar{f} \alpha_{\phi q}^{(3)} \gamma^\mu \tau^j L f + \text{H.c.}, \\ & i\phi^\dagger D_\mu \phi \bar{f} \alpha_{\phi u} \gamma^\mu \tau^u R f + \text{H.c.}, \\ & i\phi^\dagger D_\mu \phi \bar{f} \alpha_{\phi d} \gamma^\mu \tau^d R f + \text{H.c.}, \\ & \frac{1}{\sqrt{2}} \phi^\dagger \tau^2 D_\mu \phi \bar{f} \alpha_{\phi \phi} \gamma^\mu \tau^- R f + \text{H.c.}, \\ & -\phi^\dagger \phi \tilde{f} \alpha_{u\phi} R u + \text{H.c.}, \\ & -\phi^\dagger \phi \bar{f} \alpha_{d\phi} R d + \text{H.c.}, \end{aligned} \quad (113)$$

where

$$D_\mu \phi = \left(\partial_\mu + ig \frac{\tau}{2} \cdot W_\mu + i \frac{g'}{2} B_\mu \right) \phi. \quad (114)$$

The coefficients appearing in Eq. (113) take the values

$$\begin{aligned} \alpha_{\phi q}^{(1)} &= 0, \\ \alpha_{\phi q}^{(3)} &= 0, \end{aligned} \quad (115)$$

$$(\alpha_{\phi u})_{ij} = -\frac{1}{2} \lambda_i^{(u)\dagger} \lambda_j^{(u)} \frac{1}{M^2},$$

$$(\alpha_{\phi d})_{ij} = \frac{1}{2} \lambda_i^{(d)\dagger} \lambda_j^{(d)} \frac{1}{M^2},$$

$$(\alpha_{\phi \phi})_{ij} = \frac{1}{2} \lambda_i^{(u)\dagger} \lambda_j^{(d)} \frac{1}{M^2},$$

$$\tilde{y}_u \rightarrow \tilde{y}_u (I - \alpha_{\phi u} M^2),$$

$$\tilde{y}_d \rightarrow \tilde{y}_d (I + \alpha_{\phi d} M^2).$$

The above results are taken from [19] and have been derived in a linear realization of the symmetry group, where the Higgs field h is explicitly included, along with the Goldstone bosons. It is easy however to recover the leading contribution to the coefficients of our effective operators (8). The procedure would amount to integrating out the Higgs field, of course. This would lead to two type of contributions: tree-level and one loop. The latter are enhanced by logs of the Higgs boson mass, but suppressed by the usual loop factor $1/16\pi^2$. In addition there are the multiplicative Yukawa couplings. It is not difficult to see though that only the light fermion Yukawa couplings appear and hence the loop contribution is small. To retain the tree-level contribution only we simply replace ϕ by its vacuum expectation value.

Since $\alpha_{\phi q}^{(1)}$ and $\alpha_{\phi q}^{(3)}$ are zero there is no net contribution to the left effective couplings. On the contrary, $\alpha_{\phi u}$, $\alpha_{\phi d}$, and $\alpha_{\phi \phi}$ contribute to the effective operators containing right-handed fields

$$\begin{aligned} \frac{M_R^{2\dagger} + M_R^{2\dagger}}{2} &= -\frac{v^2}{8} (\alpha_{\phi d} + \alpha_{\phi d}^\dagger + \alpha_{\phi u} + \alpha_{\phi u}^\dagger), \\ \frac{M_R^{2\dagger} - M_R^{2\dagger}}{2} &= \frac{v^2}{8} (\alpha_{\phi \phi} - \alpha_{\phi \phi}^\dagger), \end{aligned} \quad (116)$$

$$\frac{M_R^{1\dagger} + M_R^{1\dagger}}{2} = \frac{v^2}{16} (\alpha_{\phi d} + \alpha_{\phi d}^\dagger - \alpha_{\phi u} - \alpha_{\phi u}^\dagger + \alpha_{\phi \phi} + \alpha_{\phi \phi}^\dagger),$$

$$\frac{M_R^{3\dagger} + M_R^{3\dagger}}{2} = \frac{v^2}{16} (\alpha_{\phi d} + \alpha_{\phi d}^\dagger - \alpha_{\phi u} - \alpha_{\phi u}^\dagger - \alpha_{\phi \phi} - \alpha_{\phi \phi}^\dagger).$$

As we can see, the contribution to the effective couplings, and hence to the observables, is always suppressed by a power of M^{-2} , the scale of the new physics, as announced in the introduction. The contribution from many other models involving heavy fermions can be deduced from [19] in a similar way and general patterns inferred.

The second example we would like to briefly discuss is the standard model itself. Particularly, the standard model in the limit of a heavy Higgs boson. In the case without mixing the effective coefficients were derived in [5]. The results in the general case where mixing is present are given by

$$\begin{aligned}
(\tilde{M}^2 - \tilde{M}^{2\dagger})_{ij} &= -\frac{1}{16\pi^2} \frac{m_i^u K_{ij} m_j^d - m_i^d K_{ij}^\dagger m_j^u}{4v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
(\tilde{M}^2 + \tilde{M}^{2\dagger})_{ij} &= \frac{1}{16\pi^2} \frac{m_i^{d2} - m_i^{u2}}{4v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right) \delta_{ij}, \\
(\tilde{M}^1 + \tilde{M}^{1\dagger})_{ij} &= -\frac{1}{16\pi^2} \frac{(m_i^{u2} + m_i^{d2}) \delta_{ij} + m_i^u K_{ij} m_j^d + m_i^d K_{ij}^\dagger m_j^u}{8v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
(\tilde{M}^3 + \tilde{M}^{3\dagger})_{ij} &= -\frac{1}{16\pi^2} \frac{(m_i^{u2} + m_i^{d2}) \delta_{ij} - m_i^u K_{ij} m_j^d - m_i^d K_{ij}^\dagger m_j^u}{8v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
(N^4 + N^{4\dagger})_{ij} &= \frac{1}{16\pi^2} \frac{m_i^{u2} \delta_{ij} - K_{ik} m_k^{d2} K_{kj}^\dagger}{4v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{1}{2} \right), \\
(N^{2\dagger} + N^2)_{ij} &= \frac{1}{16\pi^2} \frac{m_i^{u2} \delta_{ij} - K_{ik} m_k^{d2} K_{kj}^\dagger}{4v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
(N^1 + N^{1\dagger})_{ij} &= -\frac{1}{16\pi^2} \frac{m_i^{u2} \delta_{ij} + K_{ik} m_k^{d2} K_{kj}^\dagger}{4v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
(N^3 + N^{3\dagger})_{ij} &= 0, \\
(N^2 - N^{2\dagger})_{ij} &= -(N^4 - N^{4\dagger})_{ij},
\end{aligned} \tag{117}$$

where we have used dimensional regularization with $d=4-2\epsilon$ and $\{\gamma^5, \gamma^\mu\}=0$; we have also defined $1/\hat{\epsilon}=1/\epsilon-\gamma+\log 4\pi$. From Eqs. (117), (41) and (42) we immediately obtain the contribution to the Z and W current vertices

$$\begin{aligned}
g_L^u &= \frac{1}{16\pi^2} \frac{m_i^{u2} \delta_{ij}}{2v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
g_L^d &= -\frac{1}{16\pi^2} \frac{m_i^{d2} \delta_{ij}}{2v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
g_R^u &= -\frac{1}{16\pi^2} \frac{m_i^{u2} \delta_{ij}}{2v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
g_R^d &= \frac{1}{16\pi^2} \frac{m_i^{d2} \delta_{ij}}{2v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
h_L &= \frac{1}{16\pi^2} \frac{m_i^{u2} K_{ij} + K_{ij} m_j^{d2}}{4v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right), \\
h_R &= -\frac{1}{16\pi^2} \frac{m_i^u K_{ij} m_j^d}{2v^2} \left(\frac{1}{\hat{\epsilon}} - \log \frac{M_H^2}{\mu^2} + \frac{5}{2} \right).
\end{aligned} \tag{118}$$

These coefficients summarize the non-decoupling effects of a heavy Higgs boson in the standard model. Note that a heavy Higgs boson gives rise to radiative corrections that do not contribute to flavor changing neutral currents, but generates contributions to the charged currents that alter the unitarity of the left mixing matrix $\mathcal{U}$ and produces a right mixing matrix which is non-unitary and of course is not present at tree level.

The divergence of these coefficients just reflect that the Higgs boson is a necessary ingredient for the standard model to be renormalizable. These divergences cancel the singularities generated by radiative corrections in the light sector. At the end of the day, this amounts to cancelling all $1/\epsilon$ and replacing $\mu \rightarrow M_W$.

Although, strictly speaking, the above results hold in the minimal standard model, experience from a similar calculation (without mixing) in the two-Higgs doublet model [20] leads us to conjecture that they also hold for a large class of extended scalar sectors, provided that all other scalar particles in the spectrum are made sufficiently heavy. Unless some additional CP violation is included in the two-doublet potential, there is only one phase: the one of the standard model.

Thus we have seen how different type of theories lead to a very different pattern for the coefficients of the effective theory and, eventually, to the CP-violating observables. Theories with scalars are, generically, non-decoupling, with

large logs, which are nevertheless suppressed by the usual loop factors. Theories with additional fermions are decoupling, but provide contributions already at tree level. For heavy doublets only in the right-handed sector, it turns out.

VII. CONCLUSIONS

In this work we have performed a rather detailed analysis of the issue of possible departures from the standard model in the charged current sector, with an special interest in the issue of possible new sources of CP violation. The analysis we have performed is rather general. We only assume that all—so far—undetected degrees of freedom are heavy enough for an expansion in inverse powers of their mass to be justified.

We have retained in all cases the leading contribution to the observables from the effective Lagrangian. To be fully consistent one should, at the same time, include the one-loop corrections from the standard model without a Higgs boson (universal). We have not done so, so our results are sensitive to the contribution from the new physics—encoded in the coefficients of the effective Lagrangian—inasmuch as this dominates over the standard model radiative corrections. Anyhow, it is usually possible to treat radiative corrections with the help of effective couplings, thus falling back again in an effective Lagrangian treatment.

There are two main theoretical results presented in this work. First of all, we have performed a complete study of all the possible new operators, to leading order, and the way to implement the passage to the physical basis when these additional interactions are included. To our knowledge this is the first time that this issue has been considered in the present framework with such an exhaustive detail. Secondly, we have analyzed in detail the issue of wave function and CKM matrix element renormalization. Both need to be included when the contribution from the effective operators to the different observables is considered. This has been, to our knowledge, been ignored in past treatments in the literature. As mentioned in the paper, the issue is interesting by itself.

We have also computed the relevant coefficients in a number of theories. Theories with extended matter sectors give, in principle, relatively large contributions, since they contribute at the tree level. When only heavy doublets are considered, the relevant left couplings are left untouched. Observable effects should be sought after in the right-handed sector. The contribution from the new physics is decoupling (i.e. vanishes when the scale is sent to infinity). However the limits on additional vector generations are weak, roughly one requires only their mass to be heavier than the top one, so this may lead to large contributions. Of course, there are mixing parameters λ , which can be bound from flavor changing phenomenology. Measuring the right-handed couplings seems the most promising way to test these possible effects. Stringent bounds exist in this respect from $b \rightarrow s\gamma$, constraining the couplings at the few per mille level [21]. If one assumes some sort of naturalness argument for the scale of the coefficients in the effective Lagrangian, this precludes observation unless at least the 1% level of accuracy is reached. Theories with extended scalar sectors are (unless

fine tuning of the potential is present such as in e.g. supersymmetric theories) non-decoupling and in order to make its contribution larger than the universal radiative corrections one requires a heavy Higgs boson (although their contribution, with respect to universal radiative corrections is nevertheless enhanced by the top Yukawa coupling).

In general, even if the physics responsible for the generation of the additional effective operators is CP -conserving, phases which are present in the Yukawa and kinetic couplings become observable. This should produce a wealth of phases and new CP -violating effects. As we have seen, contributions reaching the 1% level are not easy to find, so it will be extremely difficult to find any sizeable deviation with respect the standard model in the ongoing experiments.

A systematic study of the phenomenology of these couplings is now under way, as is clear that a lot of work remains to be done, such as identifying the adequate observables for the wealth of phases that might appear. Furthermore, we have obtained the effective Lagrangian at the M_W scale and we still have to scale down to the b , c or kaon mass, which is a nontrivial task.

On a more practical level our results are relevant on three different fronts. First of all we have, hopefully, clarified the issue of wave-function and CKM matrix elements renormalization. While the use we have made of our proposal is limited (only one coefficient of the effective Lagrangian contributes to the wave function and CKM renormalizations), our proposal meets all the necessary requirements. Secondly, we can incorporate a good part of the radiative corrections in the standard model itself in the $d=4$ effective operators (we have seen that explicitly for the Higgs contribution) so our results will be relevant the day that experiments become accurate enough so that radiative corrections are required. Finally, the effective Lagrangian approach consists not only in writing down the Lagrangian itself, but it comes with a well defined set of counting rules. This set of counting rules allows in the case of the CKM matrix elements a perturbative treatment of the unitarity constraint. If one assumes that the contribution from new physics and radiative corrections are comparable, then it is legitimate to use the unitarity relations in all one-loop calculations. On the contrary the tree-level predictions should be modified to account for the presence of the new-physics which introduces new phases. This procedure can be extended to arbitrary order.

Note added in proof. We recently became aware of yet another modification of the Aoki *et al.* and Denner and Sacks renormalization conditions by Barroso, Brucher and Santos [22] which apparently leads to gauge independent results.

ACKNOWLEDGMENTS

J.M. acknowledges financial support from Generalitat de Catalunya, grant 1998FI-00614. Financial support from grants AEN98-0431, 2000SGR 00026 and EURODAPHNE is greatly appreciated. G. D'Ambrosio, C.P. Burgess, F. del Aguila, M. Perez Victoria, M.J. Herrero and J. Matias are thanked for many discussions.

- [1] T. Appelquist and C. Bernard, Phys. Rev. D **22**, 200 (1980); A. Longhitano, *ibid.* **22**, 1166 (1980); Nucl. Phys. **B188**, 118 (1981); R. Renken and M. Peskin, *ibid.* **B211**, 93 (1983); A. Dobado, D. Espriu, and M.J. Herrero, Phys. Lett. B **255**, 405 (1991).
- [2] E. D'Hoker and E. Farhi, Nucl. Phys. **B248**, 59 (1984); **B248**, 77 (1984).
- [3] Yellow Report, CERN 93-05, edited by D. Bardin, W. Hollik, and G. Pasarino.
- [4] T. Appelquist, M. Bowick, E. Cohler, and A. Hauser, Phys. Rev. D **31**, 1676 (1985).
- [5] E. Bagan, D. Espriu, and J. Manzano, Phys. Rev. D **60**, 114035 (1999).
- [6] D. Espriu and J. Manzano, hep-ph/0004187.
- [7] T. Appelquist and J. Carazzone, Phys. Rev. D **11**, 2856 (1975).
- [8] A. Dobado, M.J. Herrero, and S. Penaranda, hep-ph/0002134.
- [9] M. Peskin and D. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Redwood City, CA, 1995).
- [10] C.P. Burgess, S. Godfrey, H. Konig, D. London, and I. Maksymyk, Phys. Rev. D **49**, 6115 (1994); E. Malkawi and C.P. Yuan, *ibid.* **50**, 4462 (1994); **52**, 472 (1995).
- [11] M. Veltman, Nucl. Phys. **B123**, 97 (1977); M. Martinez, R. Miquel, L. Rolandi, and R. Tenchini, Rev. Mod. Phys. **71**, 575 (1999).
- [12] Y. Achiman, hep-ph/9812389; T.G. Rizzo, Phys. Rev. D **58**, 055009 (1998); D. Falcone and F. Tramontano, *ibid.* **61**, 113013 (2000).
- [13] D. London, hep-ph/9708476.
- [14] W.F.L. Hollik, Fortschr. Phys. **38**, 165 (1990); **34**, 687 (1986).
- [15] D. Espriu and M.J. Herrero, Nucl. Phys. **B373**, 117 (1992).
- [16] K.I. Aoki, Z. Hioki, M. Konuma, R. Kawabe, and T. Muta, Prog. Theor. Phys. Suppl. **73**, 1 (1982); A. Denner and T. Sack, Nucl. Phys. **B347**, 203 (1990).
- [17] P. Gambino, P.A. Grassi, and F. Madricardo, Phys. Lett. B **454**, 98 (1999).
- [18] C. Balzereit, T. Mannel, and B. Plumper, Eur. Phys. J. C **9**, 197 (1999).
- [19] F. del Aguila, M. Perez-Victoria, and J. Santiago, J. High Energy Phys. **09**, 011 (2000).
- [20] P. Ciafaloni and D. Espriu, Phys. Rev. D **56**, 1752 (1997).
- [21] CLEO Collaboration, T. Swarnicki, in Proceedings of the 1998 International Conference on HEP, Vancouver, British Columbia, Canada, 1998, Vol. 2, p. 1057.
- [22] A. Barroso, L. Brucher, and R. Santos, Phys. Rev. D **62**, 096003 (2000); S. Oliveira, L. Bruecher, R. Santos, and A. Barroso, hep-ph/0011324.